



# Peter Borwein Revisited

A brother's retrospective  
prepared for

## The **Mathematical Interests** of **Peter Borwein**

**Number Theory, Analysis,  
Combinatorics,  
Computational Mathematics,  
and Mathematical Modelling**

A conference celebrating  
Peter Borwein's 55th  
Birthday and his  
contributions to  
mathematics: the  
research and the  
community



(May 14) May 12–16 , 2008  
The **IRMACS Centre**, SFU  
Burnaby, BC, Canada



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and Mathematical Modelling



A conference celebrating Peter Borwein's 55th birthday  
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the research and the community

May 12th – 16th, 2008

The IRMACS Centre, Simon Fraser University  
Burnaby, British Columbia, Canada

Registration deadline: May 1st, 2008



Website: <http://conferences.irmacs.sfu.ca/borwein08/>

Let  $q$  be an integer,  $|q| > 1$ , and let  $c$  be a rational number,  $c \neq 0$  and  $c \neq -q^n$  ( $n \in \mathbb{N}$ ). Then both

$$\sum_{n=1}^{\infty} 1/(q^n + c)$$

and

$$\sum_{n=1}^{\infty} (-1)^n / (q^n + c)$$

are irrational and not Liouville.



BBP in acrylic



# ADHD Summary

- Peter does *really* good Math(s)
- Peter *really* likes Pictures
- Peter *really* knows a lot about Pi, Pade, Polynomials and other P's
- Peter *really* is now  $55=110111$ 
  - if you wish you can now return to own research



# Outline of Talk

1. **Peter** a Chronology • 1953 until now in tranches  
– plus a math culture quiz
2. Some Mathematics • Representative high-spots
3. “Meta”-mathematics • The big picture
4. Concluding Remarks • Other things I meant to say



# This is your life 1953-1957

- **May 10** Peter is born in St Andrews, Scotland
  - he apologizes with a trike
- I am taller. He is cuter.
  - nursery teacher asks “Does he ever misbehave?”
- **1954-57** We share a bed
  - (end-to-end) I get wet
- **1955** We both get **polio**?



2.5 years and 6 months

from Lithuania to “Tel-Aviv”





# This is your life 1957-1958

Peter carefully considers various career alternatives





# St Andrews ...



Peter liked donkeys but hated sand

University (1412)

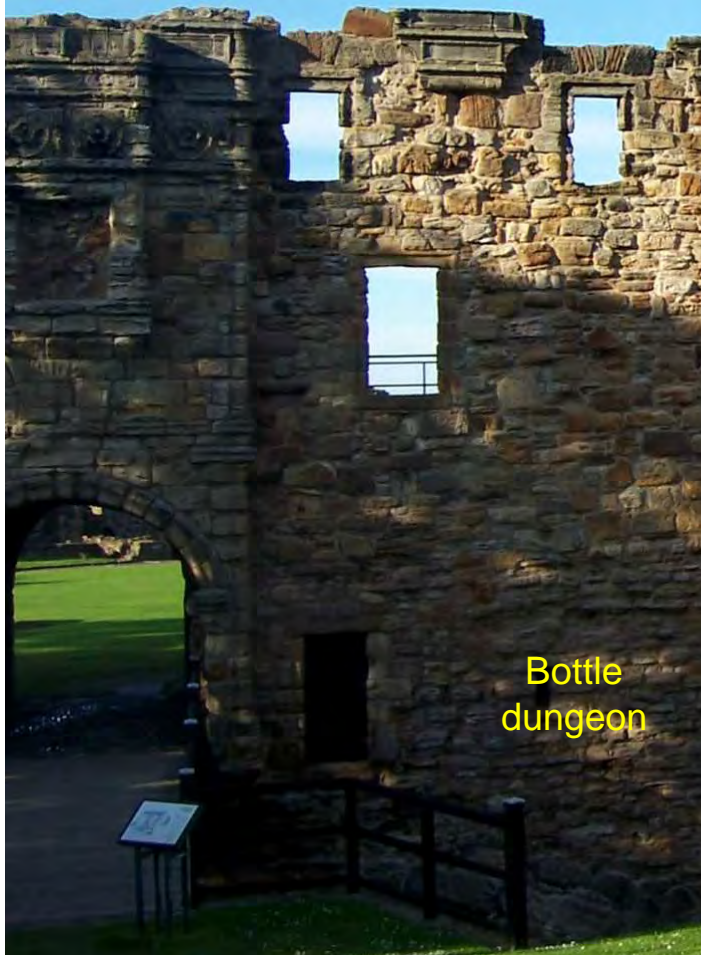
Town and Gown

St Salvator's College





# ..St Andrews oozing history



Bottle  
dungeon



Castle and pier





# This is your life 1953-1957



- He was IMU President (1955-58)
- **Heinz Hopf** (1894-1971)
- *“one of the most distinguished mathematicians of the twentieth century. His work is closely linked with the emergence of algebraic topology.”*

# This is your life 1958-1962

- **1958** Peter starts school
  - at a **Madras** (a)
  - doubles Jewish student population
- He attends his first **International Math Congress**
  - **58** in Edinburgh
  - he learns all about **Pontryagin**
- David Borwein is Secretary of the GA
  - in St Andrews
- **59** make quarantine history
- **61** Sister Sarah arrives





# This Is Your Life 1959

- Quarantined at home with scarlet fever
  - first not to spend 40 days in a fever ward
  - Bruce Shawyer taught us statistics



# This is your life 1960

We learn how to handle graduate students early

Thanks to Bessie, we usually had odder pets



## Xenopus Toad

- Carniverous
- First cloned reptile
- Often in space





# This is your life 1958-1962



- He was IMU President (1959-62)
- **Rolf Nevanlinna** (1895-1980)
- *“Nevanlinna theory appeared in a 100 page paper in 1925. ... described by Weyl as one of the few great mathematical events of our century. .”*



# This is your life 1963-1972

- **Sept 63** Peter moves to London Ontario
  - we fully experience the 60s
  - Peter spends years being made vulnerable to **tuberculosis**
  - **1967** Peter goes West
  - David Suzuki & David Shore went to Central High too
- **1971** PBB starts at UWO
  - he lives with a future CBC Morningside **producer**, an ex **porn publisher**, and a large **python**
- **1972** we travel in Europe “**on ten dollars a day**”



1969 self-portrait

Peter had a really dark room and a horn





# This is your life 2006



# This Is Your Family 1966





# This is your life 1963-1972



- He was IMU President (1967-1970)
- **Henri Cartan** (1904- )
- *“mathematicians throughout the world will welcome the availability of the 'Oeuvres' of a mathematician whose writing and teaching has had such an influence on our generation.*
  - **Remert and Serre**
- An original Bourbaki



# This is your life 1973-1982



- **73** Peter meets Jenny
- **74** they go to UBC
  - David and I warned him not to!
- **76** MSc with John Fournier
- **78** we travel in Europe
- **79** PhD with Dave Boyd
  - spends PDF year in Oxford
- **1980** joins Dalhousie
  - after *unusual* interview at UofA
  - and *untimely* job offer at Dal
- **80** marries on US Thanksgiving



Sophie, Jenny, Peter, Alex, Tessa





# This is your life 1974 at UWO



# This Is Your Wife 1975

- Outdoors and heights always attracted
  - early morning did not
- As kids a big treat was to be allowed to camp and freeze in the garden
  - in a one-man tent that had been to the top of K2 in the early 1950s
- As a researcher, walking and rolling play a big role. Also
  - caffeine
  - baths
  - notebooks





# This is your life 1973-1982



- He was IMU President (1979-82)
- **Lenhart Carleson** (1928- )
- **1966** proved Luzin's 1913 conjecture that the Fourier series of  $f$  in  $L^2(T)$  converges a.e to  $f$ .
- ***"The most important aspect in solving a mathematical problem is the conviction of what is the true result. Then it took 2 or 3 years using the techniques that had been developed during the past 20 years or so. ."***
  - 2006 Abel Prize



# Carleson's 1966 ICM address

*the problem of course presents itself already when you are a student and I was thinking about the problem on and off, but the situation was more interesting than that. The great authority in those days was Zygmund and he was completely convinced that what one should produce was not a proof but a counter-example. When I was a young student in the United States, I met Zygmund and I had an idea how to produce some very complicated functions for a counter-example and Zygmund encouraged me very much to do so. I was thinking about it for about 15 years on and off, on how to make these counter-examples work and the interesting thing that happened was that I realised why there should be a counter-example and how you should produce it. I thought I really understood what was the background and then to my amazement I could prove that this "correct" counter-example couldn't exist and I suddenly realised that what you should try to do was the opposite, you should try to prove what was not fashionable, namely to prove convergence. **The most important aspect in solving a mathematical problem is the conviction of what is the true result.** Then it took 2 or 3 years using the techniques that had been developed during the past 20 years or so. . ."*



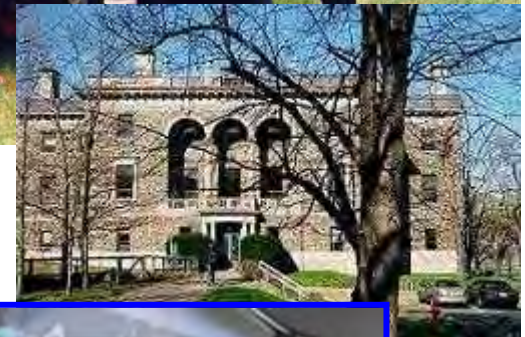
# This is your life 1983-1992

- **1982-91** we work together at Dalhousie (then 93-03 at SFU)
  - talk math for first time in lives
  - make up for lost time
- **86** Jenny becomes MD
- **86** First book appears
  - “Pi and the AGM”
- **87** 1/3 brother pairs to lecture at Ramanujan Centennial
  - says Ramanujan “idiot savant”?
- **87** Alex, **89** Sophie, and **93** Tessa arrives
- **88** organized PR for Dal strike
  - “Building a better university sorry for the inconvenience”

## Family Pyramid from 1997



Math Dept at Dalhousie



# This is your life 1983-1992



- He was IMU President (83-86)
- **Juergen Moser** (1928-1999)
- **1998** Wolf prize *“For his work on stability of dynamical systems and ... PDES”* (KAM).
  - married Gertrude Courant
  - directed both Courant and MRI at ETH
  - first picture I knew and first not by Halmos
- *“... we realised that he was very special, a prince among men, a knight in shining armour. He had all the German virtues: devotion to hard work, a love of the outdoors, a love of beauty, of music ... He was exceedingly good company to do things with, like hiking in the mountains. ... He loved adventure and to test his powers; he had great self-confidence.”* (Peter Lax)





# This is your life 1993-2002

- **1993** Peter comes to SFU
  - **93** Chauvenet & Hasse prizes (“[Ramanujan and Pi](#)”)
  - Peter fluent in Hungarian (“Bor”)
  - we start to build CECM →
  - **96** BC-CUFA faculty member of the year
  - **97** Stars on [MSNBC](#) Thanksgiving
  - **99** UWO National Alumni Merit Award ([family!](#))
  - **02** Ford prize and Jubilee medal (“[Visible structures in number theory](#)”)

One of Peter's first skills



# This is your life 1993-2002

Globe and Mail

- **1995** first URL in Canadian newspaper

- BBP algorithm
- 10,000 downloads
- finalist for \$100K

Edge of Computation Science Prize 2005

- **99** chair NSERC GSC
- **2001-02** Starts to dream up IRMACS

- demanded un-sexy acronym

## Math professor figures kid can make pi history

Canadian's guide will allow calculations to 100-billionth digit

BY STEPHEN STRAUSS  
Science Reporter

If you are willing to give up doing anything else on your home computer for a few days, Canadian and U.S. mathematicians have figured out techniques for you to make pi history.

"I predict some smart kid working on a home computer will use these methods to compute the 100-billionth digit of pi in the very near future," said Peter Borwein, a mathematician at Simon Fraser University in Burnaby, B.C.

### Breaking the pi barrier

In order to calculate pi on your computer, you need this equation and to consult Peter Borwein's World Wide Web page on the Internet. His address is <http://www.cecm.sfu.ca/~pborwein/> and look under the link Computing Pi and Related Matters.

$$\pi = \sum_{n=0}^{\text{infinity}} 16^{-n} \left( \frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right)$$

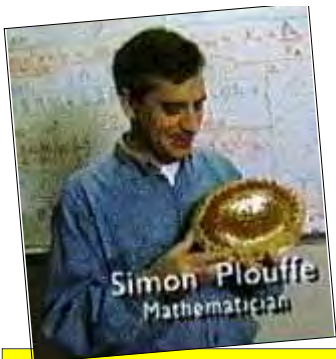
when they haven't computed all the numbers in between. For example, they know that pi to the 40-billionth place ends in the number 1.

true with his and his brother's pi world record.

There may also be a more theoretical use for the computer formula that skips the in-be-







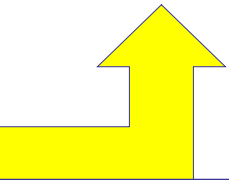
# PSLQ and Hex Digits of Pi

Finalist for the \$100K **Edge of Computation** Prize won by David Deutsch (Quantum Computing)

$$\log 2 = \sum_{n=1}^{\infty} \frac{1}{k 2^k}$$



My brother made a formula allows one to compute binary digits of log 2 without



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## THE \$100,000 EDGE OF COMPUTATION SCIENCE PRIZE

**For individual scientific work, extending the computational idea, performed, published, or newly applied within the past ten years.**

The Edge of Computation Science Prize, established by Edge Foundation, Inc., is a \$100,000 prize initiated and funded by science philanthropist Jeffrey Epstein.

# This is your life 1993-2002



- Abel Prize Banquet 2007: L. Lovász (2007-10), L. Carleson (1979-82), L. Fadeev (1987-90), Sir John Ball (2003-06)
- IMU President (1993-1996)
- David Mumford (1937- )

Abel prize 08 Thompson, Tits 07 Varadhan  
05 Lax 04 Atiyah-Singer 03 Serre





# This is your life 2003-2008+

- We've *finished* the IMU
- **2004** IRMACS funded
  - Peter made Burnaby Mountain Professor
  - reveals MS
- **2005** IRMACS exists
  - Peter designed a huge amount of it
  - MITACS/MOCAA turns 10
- **2006**      **Borwein Prize**
- **2009**      **L'avenir**

“Alex and some guy (2007)”



# Madelung's Constant: David Borwein CMS Career Award



$$= \sum'_{n,m,p} \frac{(-1)^{n+m+p}}{\sqrt{n^2 + m^2 + p^2}}$$

This polished solid silicon bronze sculpture is inspired by the work of David Borwein, his sons and colleagues, on the **conditional series** above for salt, **Madelung's constant**. This series can be summed to uncountably many constants; one is **Madelung's constant** for **electro-chemical stability of sodium chloride**. (**Convexity is hidden here too!**)

This constant is a period of an elliptic curve, a real surface in four dimensions. There are uncountably many ways to imagine that surface in three dimensions; one has negative gaussian curvature and is the tangible form of this sculpture. (**As described by Ferguson.**)



## Centre seen as 'serious nirvana'

April 07, 2005 , vol. 32, no. 7

By Carol Thorbes

Move over creators of Max Head-room, Matrix and Metropolis. What researchers can accomplish at Simon Fraser University's IRMACS centre rivals the high tech feats of the most memorable futuristic films.

The \$14 million centre's acronym stands for interdisciplinary research in the mathematical and computational sciences. The centre's expansive view of the Lower Mainland from atop Bumaby Mountain echoes its limitless potential as a facility dedicated to fostering interdisciplinary research among scientists whose primary lab tool is the computer.



*SFU mathematician and IRMACS executive director Peter Barwein (left) communicates with IRMACS collaboration and visualization coordinator Brian Carrie. To the right of them another plasma display portrays a 3D image of a molecular structure.*

A newly constructed 2,500 square metre space atop the applied sciences building, the centre has eight labs, five meeting rooms and a presentation theatre, seating up to 100 people. They are equipped with easily upgradeable computational, multimedia, internet and remote conferencing (including satellite) technology. High performance distributed computing and clustering technology, designed at SFU, and access to WestGrid, an ultra high-speed, interprovincial network, with shared computing and multimedia resources, make IRMACS unique in Western Canada.

# Some Mathematics

Very little --- since there is plenty of it elsewhere at this meeting

His own favourite work includes

- *Computational Excursions in Analysis and Number Theory*, Springer, 2002.
  - “a wonderful overview of one of the most beautiful and active areas of current computational number theory.” Sergei Konyagin, *Math Reviews*
- Mahler Measure Problems
  - Suppose  $p(x)$  is a non-cyclotomic irreducible monic polynomial of degree  $n$  and has all odd coefficients. Then the Mahler measure of  $p$  is at least 1.49
  - PBB, Dobrowolski and Mossinghoff, Lehmer's problem for polynomials with odd coefficients, *Annals of Math*, (2007).
- Bailey, PBB and Plouffe, On the rapid computation of various polylogarithmic constants, *Math. Comp* (1997).
- PBB and Erdelyi, Generalizations of Muntz's theorem via a Remez type inequality for Muntz spaces, *J. Amer. Math. Soc.* (1997).
- PBB, On the irrationality of certain series, *Proc. Cam Phil. Soc.* (1992).
- JMB, PBB, & Bailey, Ramanujan, modular equations, and approximations to pi, or How to compute one billion digits of pi, *MAA* (1989).
- JMB and PBB, Ramanujan and pi, *Scientific American* (1988).
- PBB, The Desmic conjecture, *J. Combin. Theory Ser. A* (1983).





Pi to 1.5 trillion places in 20 steps

This fourth order algorithm was used on all big-Pi computations from 1986 to 2001

$$y_1 = \frac{1 - \sqrt[4]{1 - y_0^4}}{1 + \sqrt[4]{1 - y_0^4}}, a_1 = a_0(1 + y_1)^4 - 2^3 y_1(1 + y_1 + y_1^2) \quad y_{11} = \frac{1 - \sqrt[4]{1 - y_{10}^4}}{1 + \sqrt[4]{1 - y_{10}^4}}, a_{11} = a_{10}(1 + y_{11})^4 - 2^{23} y_{11}(1 + y_{11} + y_{11}^2)$$

$$y_2 = \frac{1 - \sqrt[4]{1 - y_1^4}}{1 + \sqrt[4]{1 - y_1^4}}, a_2 = a_1(1 + y_2)^4 - 2^5 y_2(1 + y_2 + y_2^2) \quad y_{12} = \frac{1 - \sqrt[4]{1 - y_{11}^4}}{1 + \sqrt[4]{1 - y_{11}^4}}, a_{12} = a_{11}(1 + y_{12})^4 - 2^{25} y_{12}(1 + y_{12} + y_{12}^2)$$

$$y_3 = \frac{1 - \sqrt[4]{1 - y_2^4}}{1 + \sqrt[4]{1 - y_2^4}}, a_3 = a_2(1 + y_3)^4 - 2^7 y_3(1 + y_3 + y_3^2) \quad y_{13} = \frac{1 - \sqrt[4]{1 - y_{12}^4}}{1 + \sqrt[4]{1 - y_{12}^4}}, a_{13} = a_{12}(1 + y_{13})^4 - 2^{27} y_{13}(1 + y_{13} + y_{13}^2)$$

$$y_4 = \frac{1 - \sqrt[4]{1 - y_3^4}}{1 + \sqrt[4]{1 - y_3^4}}, a_4 = a_3(1 + y_4)^4 - 2^9 y_4(1 + y_4 + y_4^2) \quad y_{14} = \frac{1 - \sqrt[4]{1 - y_{13}^4}}{1 + \sqrt[4]{1 - y_{13}^4}}, a_{14} = a_{13}(1 + y_{14})^4 - 2^{29} y_{14}(1 + y_{14} + y_{14}^2)$$

$$y_5 = \frac{1 - \sqrt[4]{1 - y_4^4}}{1 + \sqrt[4]{1 - y_4^4}}, a_5 = a_4(1 + y_5)^4 - 2^{11} y_5(1 + y_5 + y_5^2) \quad y_{15} = \frac{1 - \sqrt[4]{1 - y_{14}^4}}{1 + \sqrt[4]{1 - y_{14}^4}}, a_{15} = a_{14}(1 + y_{15})^4 - 2^{31} y_{15}(1 + y_{15} + y_{15}^2)$$

$$y_6 = \frac{1 - \sqrt[4]{1 - y_5^4}}{1 + \sqrt[4]{1 - y_5^4}}, a_6 = a_5(1 + y_6)^4 - 2^{13} y_6(1 + y_6 + y_6^2) \quad y_{16} = \frac{1 - \sqrt[4]{1 - y_{15}^4}}{1 + \sqrt[4]{1 - y_{15}^4}}, a_{16} = a_{15}(1 + y_{16})^4 - 2^{33} y_{16}(1 + y_{16} + y_{16}^2)$$

$$y_7 = \frac{1 - \sqrt[4]{1 - y_6^4}}{1 + \sqrt[4]{1 - y_6^4}}, a_7 = a_6(1 + y_7)^4 - 2^{15} y_7(1 + y_7 + y_7^2) \quad y_{17} = \frac{1 - \sqrt[4]{1 - y_{16}^4}}{1 + \sqrt[4]{1 - y_{16}^4}}, a_{17} = a_{16}(1 + y_{17})^4 - 2^{35} y_{17}(1 + y_{17} + y_{17}^2)$$

$$y_8 = \frac{1 - \sqrt[4]{1 - y_7^4}}{1 + \sqrt[4]{1 - y_7^4}}, a_8 = a_7(1 + y_8)^4 - 2^{17} y_8(1 + y_8 + y_8^2) \quad y_{18} = \frac{1 - \sqrt[4]{1 - y_{17}^4}}{1 + \sqrt[4]{1 - y_{17}^4}}, a_{18} = a_{17}(1 + y_{18})^4 - 2^{37} y_{18}(1 + y_{18} + y_{18}^2)$$

$$y_9 = \frac{1 - \sqrt[4]{1 - y_8^4}}{1 + \sqrt[4]{1 - y_8^4}}, a_9 = a_8(1 + y_9)^4 - 2^{19} y_9(1 + y_9 + y_9^2) \quad y_{19} = \frac{1 - \sqrt[4]{1 - y_{18}^4}}{1 + \sqrt[4]{1 - y_{18}^4}}, a_{19} = a_{18}(1 + y_{19})^4 - 2^{39} y_{19}(1 + y_{19} + y_{19}^2)$$

$$y_{10} = \frac{1 - \sqrt[4]{1 - y_9^4}}{1 + \sqrt[4]{1 - y_9^4}}, a_{10} = a_9(1 + y_{10})^4 - 2^{21} y_{10}(1 + y_{10} + y_{10}^2) \quad y_{20} = \frac{1 - \sqrt[4]{1 - y_{19}^4}}{1 + \sqrt[4]{1 - y_{19}^4}}, a_{20} = a_{19}(1 + y_{20})^4 - 2^{41} y_{20}(1 + y_{20} + y_{20}^2)$$

# A billion Digits of Pi

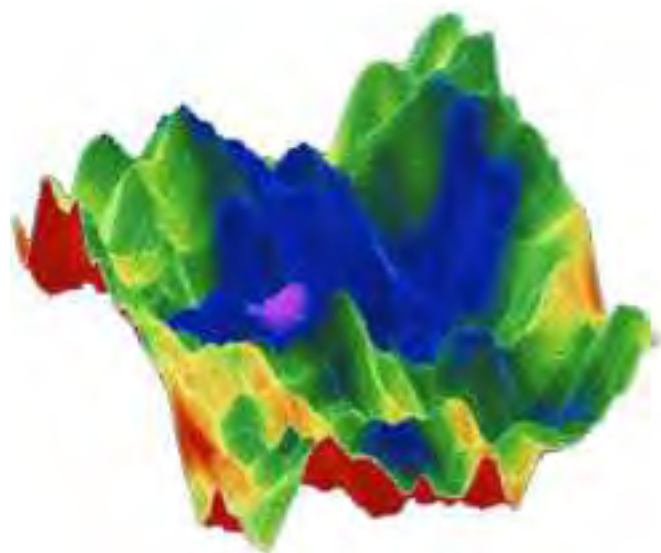
These equations specify an algebraic number:  
 $1/\pi \sim a_{20}$

Set  $a_0 = 6 - 4\sqrt{2}$  and  $y_0 = \sqrt{2} - 1$ . Iterate

$$y_{k+1} = \frac{1 - (1 - y_k^4)^{1/4}}{1 + (1 - y_k^4)^{1/4}} \quad \text{and}$$

$$a_{k+1} = a_k(1 + y_{k+1})^4 - 2^{2k+3} y_{k+1}(1 + y_{k+1} + y_{k+1}^2)$$

Then  $1/a_k$  converges quartically to  $\pi$



A random walk on a million digits of Pi

# Some Meta-Mathematics

- He has always been able to see things from a different perspective
  - examples are on next slide
  - a sensible risk taker
    - learned from parents & grandparents?
  - an early adopter (Maple2 manual)
- He doesn't waste words
  - we thought he was deaf as a kid
  - and a very quick eater
    - bones and all
  - I got really short notes in graduate student days
- He is very focussed and persistent
  - “are we done yet?”
  - but a great collaborator
  - and (too) modest

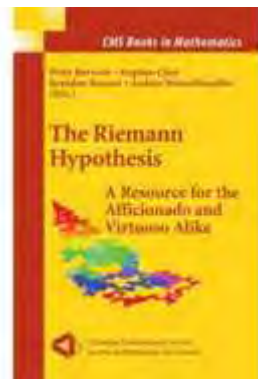




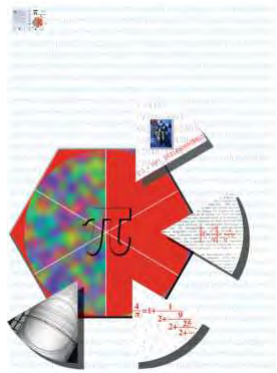
# Some Meta-Mathematics

He dreams up unusual projects

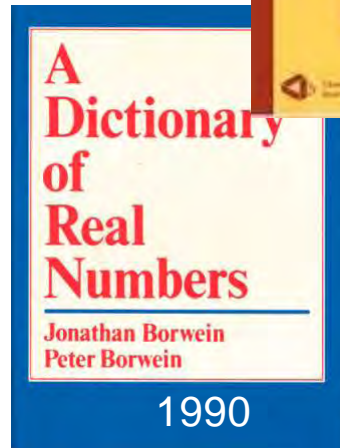
- **Inverse Calculator**
  - began as a book, ended up in Maple and as an HPC applet
- **Visual Mathematics**
  - Colour calculator
  - Ford Prize
- **Interesting Collections**



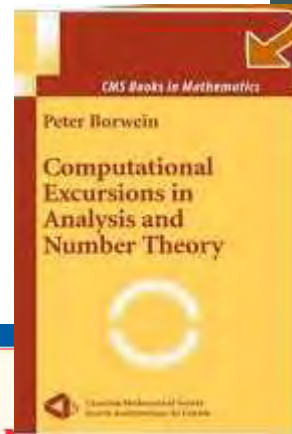
2007



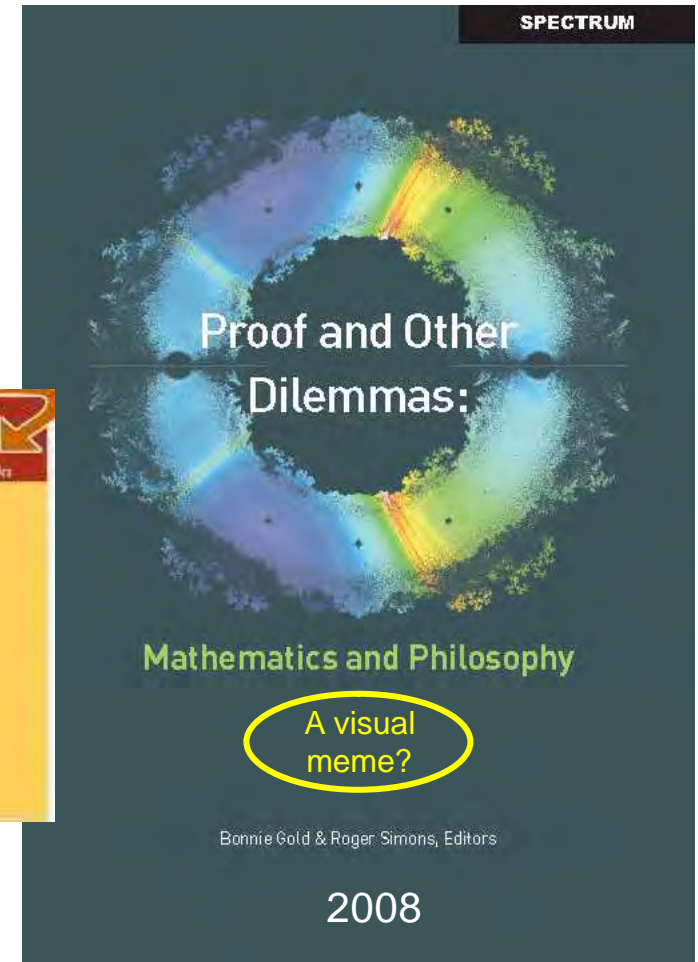
1997/2000/04



1990



This image refuses to die



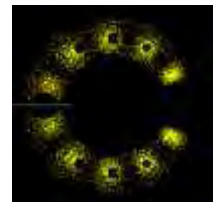
Bonnie Gold & Roger Simons, Editors

2008

A book on **Creativity** is in the works

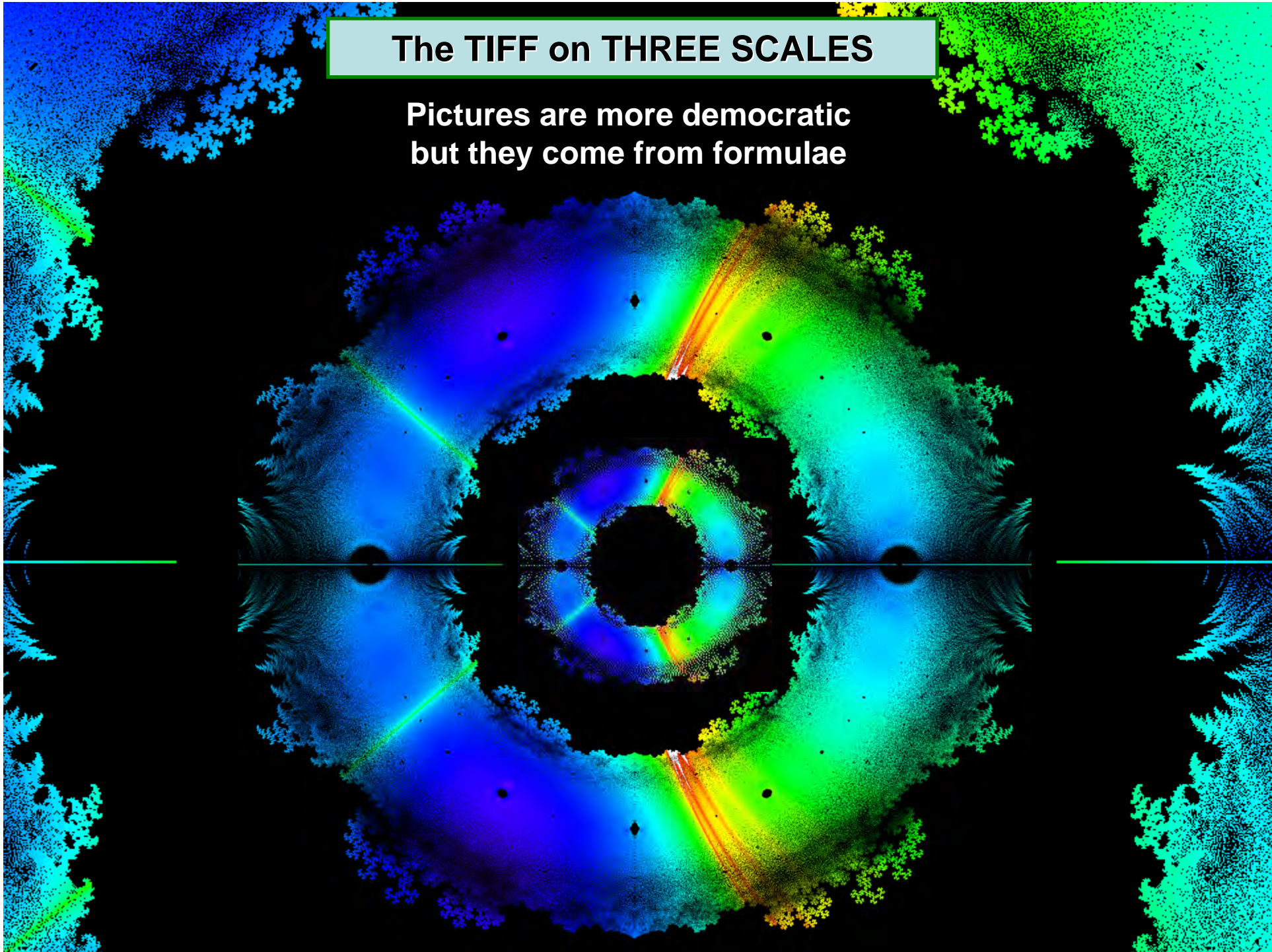


Organic Math 1995/96



# The TIFF on THREE SCALES

Pictures are more democratic  
but they come from formulae





MATH

# A Digital Slice of Pi

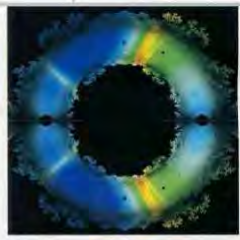
THE NEW WAY TO DO PURE MATH: EXPERIMENTALLY BY W. WAYT GIBBS

One of the greatest ironies of the information technology revolution is that while the computer was conceived and born in the field of pure mathematics, through the genius of giants such as John von Neumann and Alan Turing, until recently this marvelous technology had only a minor impact within the field that gave it birth. So begins *Experimentation in Mathematics*, a book by Jonathan M. Borwein and David H. Bailey due out in September that documents how all that has begun to change. Computers, once looked on by mathematical researchers with disdain as mere calculators, have gained enough power to enable an entirely new way to make fundamental discoveries: by running experiments and observing what happens.

The first clear evidence of this shift emerged in 1996. Bailey, who is chief technologist at the National Energy Research Sci-

entific Computing Center in Berkeley, Calif., and several colleagues developed a computer program that could uncover integer relations among long chains of real numbers. It was a problem that had long vexed mathematicians. Euclid discovered the first integer relation scheme—a way to work out the greatest common divisor of any two integers—around 300 B.C. But it wasn't until 1977 that Helaman Ferguson and Rodney W. Forcade at last found a method to detect relations among an arbitrarily large set of numbers. Building on that work, in 1995 Bailey's group turned its computers loose on some of the fundamental constants of math, such as log 2 and pi.

To the researchers' great surprise, after months of calculations the machines came up with novel formulas for these and other nat-



**COMPUTER REASONERS** of mathematical constructs can reveal hidden structure. The bands of color that appear in this plot of all solutions to a certain class of polynomials (specifically, those of the form  $1 \pm x \pm x^2 \pm x^3 \pm \dots \pm x^n = 0$ , up to  $n=18$ ) have yet to be explained by conventional analysis.

2003

**SCIENTIFIC AMERICAN** 23 brighter paper, better illustrations (including some in color) and twice as many pages for \$10 less.—*Brian Hayes*

## Serving a Silicon Master

**Mathematics by Experiment: Plausible Reasoning in the 21st Century.** Jonathan Borwein and David Bailey.  $x + 288$  pp. A K Peters, 2004. \$45.

**Experimentation in Mathematics: Computational Paths to Discovery.** Jonathan Borwein, David Bailey and Roland Girsengohn.  $x + 557$  pp. A K Peters, 2004. \$49.

Once upon a time, in ancient Greece, science was platonist and *a priori*. The Sun revolved around the Earth in a perfect circle, because the circle is such a perfect figure; there were four elements, because four is such a nice number, and so forth. Then along came Bacon, Boyle, Galileo, Kepler, Lavoisier, Newton and their buddies, and revolutionized science, making it experimental and empirical.

But math remains *a priori* and platonist to this day. Kant even went to excruciating lengths to "show" that geometry, although synthetic, is nevertheless *a priori*. Sure, all mathematicians, great and small, conducted experiments (until recently, using paper and pencil), but they kept their diaries and notebooks well hidden in the closet.

But stand by for a paradigm shift: Thanks to Its Omnipotence the Computer, math—that last stronghold of dear Plato—is becoming (overtly!) experimental, *a posteriori* and even contingent. But what are poor pure mathematicians to do? Their professional well-being—in other words, their philosophy—and more important, their working habits—in other words, their methodology—never prepared them for serving this new silicon master. Some of them, like the conceptual genius Alexander Grothendieck, even consider the computer (seriously!) the devil. But although many pure mathematicians strongly dislike and mistrust

s are eye-openers. I be *Symmetry Comes* ext, and *Embedded* tant supplementa- tist choose between note that *Symmetry* ce th

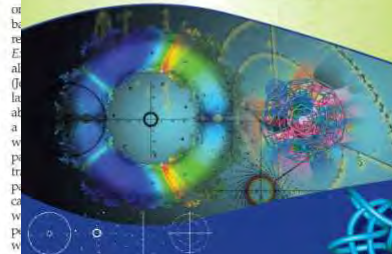
These are such fun books to read! Actually, calling them books does not do

base 2) without computing the first billion digits. It was discovered with the aid

David H. Bailey  
Jonathan M. Borwein  
Neil J. Calkin  
Roland Girsengohn  
D. Russell Lake  
Victor H. Moll

2007

# Experimental Mathematics in Action



This figure plots all roots of polynomials,  $\beta_n$ , with coefficients in  $[-1, 1]$  up to degree  $N=18$ . The zeroes are colored by their local density normalized to the range of densities, from red (low) to yellow (high). The fractal structures and holes around the roots come in different shapes and have precise locations. From *Experimentation in Mathematics*.

iveliness with their and their stories cooled by entertain- am more (include) than ngle vol- learn by erimental is a de- count of Borwein-  $\pi$   $\frac{1}{87+6}$  but the Peter. Si- rujuan, is d Inverse

to com- of  $\pi$  (in

Many people regard mathematics as the crown jewel of the sciences. Yet math has historically lacked one of the defining trappings of science: laboratory equipment. Physicists have their particle accelerators; biologists, their electron microscopes; and astronomers, their telescopes. Mathematics, by contrast, concerns not the physical landscape but an idealized, abstract world. For exploring that world, mathematicians have traditionally had only their intuition.

Now, computers are starting to give mathematicians the lab instrument that they have been missing. Sophisticated software is enabling researchers to travel further and deeper into the mathematical universe. They're calculating the number pi with mind-boggling precision, for instance, or discovering patterns in the contours of beautiful, infinite chains of spheres that arise out of the geometry of knots.

Experiments in the computer lab are leading mathematicians to discoveries and insights that they might never have reached by traditional means. "Pretty much every [mathematical] field has been transformed by it," says Richard Canoll, a mathematician at Reed College in Portland, Ore. "Instead of just being a number-crunching tool, the computer is becoming more like a garden shovel that turns over rocks, and you find things underneath."

At the same time, the new work is raising unsettling questions about how to reward experimental results

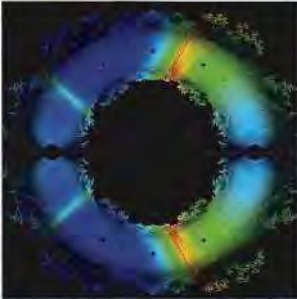
"I have some of the excitement that Leonar felt when he encountered Arabic arithmetic. I tain calculations flabbergastingly easy," Borw I think is happening with computer exper

**EXPERIMENTERS OF OLD** In one sense are nothing new. Despite their field's reputati tive science, the great mathematicians ove never limited themselves to formal reasoni For instance, in 1656, sheer curiosity and lov Newton to calculate directly the first 36 digi later writing, "I am ashamed to tell you to ha ried these computations, having no other bu Carl Friedrich Gauss, one of the toweri

tury mathema covered now n by experimenti looking for pi are a teenage experiments le most importar history of num number of prin a number  $z$  is divided by the

Gauss often experimentally prove them for plained. "I hav not yet know h In the case o theorem, Gau conjecture, but how to prove it century for mat up with a proo

"Like today' math experime century used those des, the ple with a spe



**UNSOLVED MYSTERIES** — A computer experiment produced this plot of all the solutions to a collection of simple equations in 2001. Mathematicians are still trying to account for its many features.

# MATH LAB

Computer experiments are transforming mathem

Science News BY ERICA KLARREICH

2004

THE FRONTIERS COLLECTION



# INFORMATION AND ITS ROLE IN NATURE

J. G. Roederer



Amer. Scientist 2006

2006

# Examples of Interfaces



# Pascal's Triangle Interface

INSTRUCTIONS

Output Image

Rows (max 100):

30

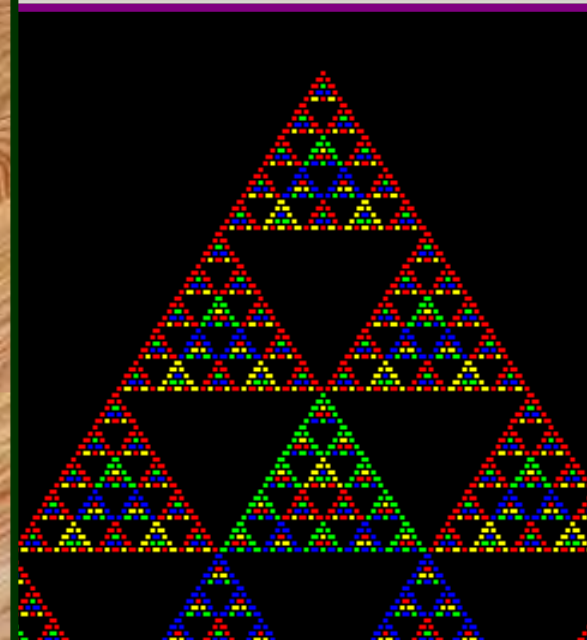
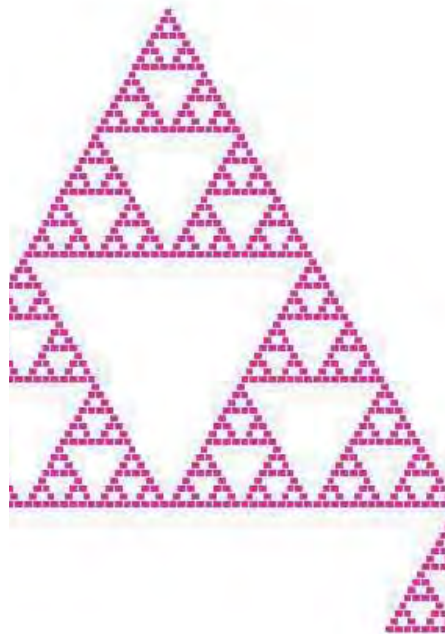
Modulus (2 to 16):

5

Image size:

300

1 1 1 **2** 1 1 3 3 1 1 **4** **6** 4 1 1 5 **10** **10** 5 1  
1 **6** 15 **20** 15 **6** 1 1 7 21 35 21 7 1



# Jon Borwein's Math Resource Portal

The following is a list of useful math tools.

## Utilities

1. [ISC2.0: The Inverse Symbolic Calculator](#)
2. [EZ Face : An interface for evaluation of Euler sums and Multiple Zeta Values](#)
3. [3D Function Grapher](#)
4. [GraPHedron: Automated and computer assisted conjectures in graph theory](#)
5. [Julia and Mandelbrot Set Explorer](#)
6. [Embree-Trefethen-Wright pseudospectra and eigenproblem](#)

## Reference

7. [The On-Line Encyclopedia of Integer Sequences](#)
8. [Finch's Mathematical Constants](#)
9. [The Digital Library of Mathematical Functions](#)
10. [The Prime Pages](#)

## Content

11. [Experimental Mathematics Website](#)
12. [Wolfram Mathworld](#)
13. [Planet Math](#)
14. [Numbers, Constants, and Computation](#)
15. [Wikipedia: Mathematics](#)

## ICCOPT 2007 Short Course

16. [Jon's Lectures](#)





The Inverse Symbolic Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.



**isc**  **inverse symbolic calculator**

Standard lookup results for **12.587886229548403854**

$\exp(1)+\pi^2$

**ISC** The original ISC

The Dev Team: Nathan Singer, Andrew Shoufdice, Lingyun Ye,  
Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

3.146264370

19.99909998

**ISC** The original ISC

The Dev Team: Nathan Singer, Andrew Shoufdice, Lingyun Ye,  
Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

The ISC presently accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

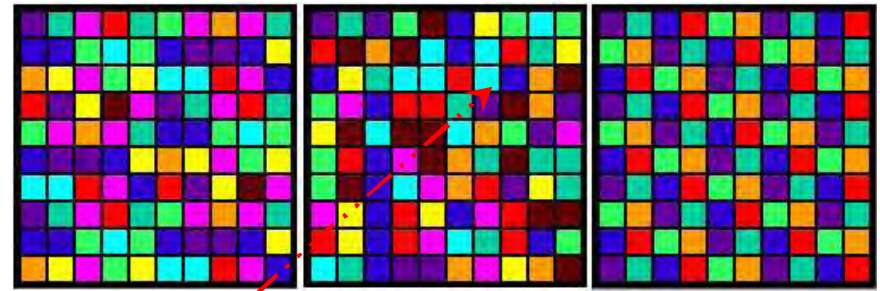
Visit:

[Jon Borwein's Webpage](#)

[David Bailey's Webpage](#)

[Math Resources Portal](#)

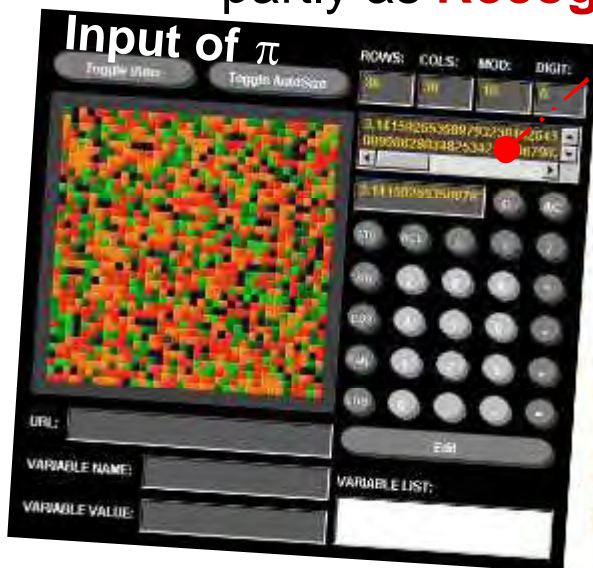
# ISC and Colour Calculators



Archimedes:  $223/71 < \pi < 22/7$

## Inverse Symbolic Computation

- ◆ mixes **large table lookup**, integer relation methods and intelligent preprocessing – needs **micro-parallelism**
  - ◆ “**Inferring symbolic structure from numerical data**”
  - ◆ faces the “curse of exponentiality”
- implemented as **identify** in Maple and partly as **Recognize** in Mathematica



identify(sqrt(2.)+sqrt(3.))

$$\sqrt{2} + \sqrt{3}$$

## INVERSE SYMBOLIC CALCULATOR

Please enter a number or a Maple expression:

Run  Clear

Simple Lookup and Browser for any number.  
 Smart Lookup for any number.  
 Generalized Expansions for real numbers of at least 16 digits.  
 Integer Relation Algorithms for any number.

Expressions that are **not** numeric like  $\ln(\pi * \sqrt{2})$  are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.



# Final Testimonials

# Kevin Hare

I guess some of the things that come to mind are how my first paper with Peter came about. We were at the ISSAC conference in Vancouver. Peter came up to me over lunch and said something to the effect of "This afternoon looks really boring, did you want to go hiking instead?" Next thing I knew, he had organized a few people to go up the grouse grind. On the way up he decided to start asking me math questions. (And I guess more importantly, I was able to ask him math questions without him getting distracted by phone calls.)

I found his approach to supervising was basically, toss lots of questions at me and hope that one of them sticks. If anybody has ever sat beside him during a conference talk, they know exactly what I mean by that. Or, I guess anybody that has been sitting with him in a pub/coffee shop while conference talks are going on also knows what I am talk about.

The only other thing I can think of really is his addiction to coffee. I remember going to a conference in Portland while he was driving us all in a mini-van. I suspect during what should have been a 6 hour drive, we stopped for coffee about 8 times. It was during this drive that he decided on the naming conventions for all of the Mitacs machines, (all after some type of coffee). Since then, he got his espresso machine in his office, and he is probably even more addicted.



# Karl Dilcher

Peter was my "official" PDF supervisor, and I believe I was his first PDF. Without his support and encouragement I would probably not be where I am now. **What I admired (and still admire) most about Peter is the fact that he always has a problem, or problems, on his mind; he will ask you, prod you, share insights with you and be very persistent. When he has found a truly exciting and worthwhile problem, he will not let go until it is solved.** He will hack away at it from all directions, will try to get others interested and involved (and often succeed in this), and more often than not he will eventually make substantial progress, either alone, or in collaboration with others.

An example for this is the spectacular BBP formula. Already during his time at Dalhousie he often said that he wanted to find the 10 billionth (I believe) digit of pi without having to know all the previous digits. I had no doubt that he would eventually succeed, and I consider BBP as a result of this (though hexadecimal instead of decimal).



Peter likes funny anecdotes, often related to mathematics or mathematicians. [One] I remember: **A visit with daughter Alex as a toddler to Point Pleasant Park, when Alex pointed to the Sailors Memorial and said, 'look, Daddy, a big plus!'**

Peter was and is an excellent writer and expositor. But there wasn't a name, usually foreign, that he wouldn't find a way to misspell. One time in the mid-90s I got a (positive) referee report in which every name was incorrect; I KNEW that Peter had to be the referee. When I gave him an off-print later on, I remarked that I thought he might already be familiar with the paper. Peter, very surprised, **"How do know I refereed it?"** I had to tell him the truth.

# Mike Mossinghoff

It's always a pleasure to visit Peter and his family and friends in "**Mahler measure heaven**", as Jeff Vaaler has called Vancouver. But one of my most memorable meetings with Peter was in Nashville, at an approximation theory meeting in the late 90's. I was just passing through, driving from N.Carolina to Texas, but my wife and I briefly crashed the conference to join Peter for lunch at a local Indian buffet. I'd visited SFU the prior summer, and we'd had some correspondence on some problems. But it was at lunch in Nashville that we hammered out plans for our first joint paper.

Working with Peter has always been just as comfortable. Since that lunch meeting, I have visited Peter some six or seven times at Simon Fraser and the CECM and IRMACS, and we now have some nine papers together.

Coming to SFU was always an exciting time. Peter seems to have an incredible knack for suggesting just the right sort of problem to his collaborators -- problems that are not only irresistible, but very well-suited to the listener's interests. He's a lot like Erdos in this respect. Each time I arrived in Vancouver, Peter always had a fascinating new project that was just irresistible to join.



## and finally

I first met Peter when he appeared in a blazer, flannel shorts, and knee high socks in my grade 6 class. That outfit never appeared again. Later he convinced me to buy a Peter Paul and Mary record and Firesign Theatre and other joys of the sixties.

He was one of the few who had a worse voice than I did and one can blame a lot of the terrible folk and experimental music that I wrote later on his influence.

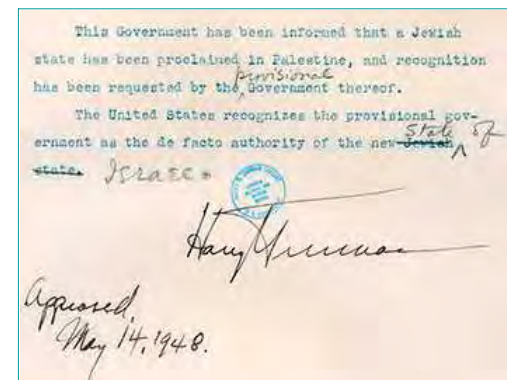
We debated the invasion of Czechoslovakia and Vietnam while our hair grew longer and I went bald while he grew smarter. He may be one of the few who has escaped Grace Slick's best line

"anyone who claims to remember the sixties wasn't there"

– I didn't.

Happy Birthday Pied Peter!

– Stephen



# It is a Dead Parrot

- Peter also speaks Monty Python on our 1996 [audio-pi applet](#)
  - which almost crashed SFU on PiDay 2003
  - and was listed as a “[best experimental music site](#)” on the web
  - so he must have a sense of humour too



King Parrot

Photo courtesy Naomi Borwein







# Conclusion

- I'm actually on sabbatical
  - I've done that
- It is *amazing* the meeting took place within the week of Peter's birthday
- *Thank you* for being here
  - Well done to everyone!
  - Especially Vezo and Pam and all at IRMACS-SFU
  - Peter deservedly seems to inspire loyalty



Special thanks to Alexandra Borwein for helping me find many pictures