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Borwein, Jonathan M. (3-SFR-EX);
Lewis, Adrian S. (3-WTRL-B)

★**Convex analysis and nonlinear optimization. (English. English summary)**

Theory and examples.

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This text is an account of convex analysis as the unifying theory underlying current computational optimization techniques. The classic text by R. T. Rockafellar [Convex analysis, Princeton Univ. Press, Princeton, N.J., 1970; MR 43#445] has recently had a more general sequel by Rockafellar and R. J.-B. Wets [Variational analysis, Springer, Berlin, 1998; MR 98m:49001]. The present book gives a concise treatment of the area, aiming to show the relevance in particular of new developments in nonsmooth analysis to optimization theory. The material is set in Euclidean spaces but in a coordinate-free style which invites extension to Banach spaces. Such an extension is discussed in the last chapter of the book. Generally there is a progression in the study of optimality conditions from an assumption of differentiability towards nondifferentiability through convexity and then local Lipschitz continuity.

There is a logical introduction of new techniques to deal with the situations of greater generality. The Karush-Kuhn-Tucker conditions for differentiable functions with differentiable inequality constraints are introduced early on. After developing appropriate theory for convex functions, the K-K-T conditions are discussed for convex functions with convex inequality constraints. The theory builds to a climax with a fuller treatment of the K-K-T theory towards the end of the book.

A feature of the presentation is that special cases of advanced techniques are introduced at an early stage. Even in the early section on the interplay between convexity and differentiability in optimality conditions, Ekeland's variational principle is proved for differentiable functions, anticipating the later full statement of the principle [I. Ekeland, J. Math. Anal. Appl. 47 (1974), 324–353; MR 49#11344] for finite-dimensional spaces used as a key to a discussion of metric regularity.

There is a fascinating interweaving of theory and applications which themselves suggest new theory. For example, after developing differentiability theory for convex functions there is a useful exposition of the duality between a convex function and its Fenchel conjugate, which in turn is significant for duality theory of convex programs. From application in convex process duality there is the introduction of the new concept of multivalued maps.

Borwein is a master in the area of nonsmooth optimization. Along with the well-known Clarke subdifferential [F. H. Clarke, Trans. Amer. Math. Soc. 205 (1975), 247–262; MR 51#3373] we meet the Michel-Penot subdifferential [P. G. Michel and J.-P. Penot, C. R. Acad. Sci. Paris Ser. I Math. 298 (1984), no. 12, 269–272; MR 85i:49027] and the limiting subdifferential of B. Sh. Mordukhovich [J. Appl. Math. Mech. 40 (1976), no. 6, 960–969 (1977); translated from Prikl. Mat. Meh. 40 (1976), no. 6, 1014–1023; MR 58#7284]. A calculus is developed for these subdifferentials and applications are sought in general nondifferentiable situations. The final chapter concerns fixed point theorems and selection theorems for multivalued maps but then applies them to study multivalued variational inequalities.

The book is written as a teaching text for beginning graduate students. It is intended for a wide audience; the student will need a certain level of mathematical maturity but no sophisticated knowledge of real analysis is assumed. Each of the 30 or so sections is about a lecture length and is followed by exercises classified as those which illustrate the ideas of the lecture and those which introduce additional theory of varying difficulty. A student working with the book as a lecture course would soon realise the importance of working the exercises; familiarity with manipulating concepts introduced in earlier sections will be necessary for full understanding and often theory refers to material introduced in earlier exercises.

The book is of manageable size and as such should appeal to the student. Further, the proofs are generally short and snappy, revealing the power of the abstract structural approach and fruitful interplay of geometrical and topological ideas. However, considerable ground is covered and, as a graduate text should, it develops the subject up to the frontiers of current research, giving an idea of areas for further exploration. Before each exercise section there is an informative guide to the relevant literature with references to a full bibliography. This text will give impetus to the teaching of analysis because it makes evident its significant applications in optimization. But it will also bring added attraction to the study of optimization because it reveals so much of its abstract structural base. *John R. Giles* (5-NEWC)